FREE VIBRATION ANALYSIS OF COMPOSITE BEAMS USING FINITE ELEMENTS AND SHEAR DEFORMATION THEORY

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Abstract - Composite materials have interesting properties such as high strength to weight ratio, ease offabrication, good electrical and thermal properties compared to metals. A laminated compositematerial consists of several layers of a composite mixture consisting of matrix and fibers. Eachlayer may have similar or dissimilar material properties with different fiber orientations undervarying stacking sequence. There are many open issues relating to design of these laminatedcomposites. The finite element software ANSYS v14.5 is used to simulate the free vibrations. The parameters investigated include the effect of fiber orientation, thickness of beam, length of beam and supportconditions. The composite beam consists of four layers of composite beam which has a total thickness of 5mm, 300 mm length and 25 mm width. The natural frequencies of analytical results and theoretical values are presented for various ply orientation angle and for various boundary conditions of different composite material, and the mode shapes are also determined using ANSYS 14.5. Then the comparisons of shear deformation and ANSYS results have obtained a good agreement.
Keywords: ANSYS 14.5, composite beams, free vibration, mode shapes, natural frequencies, shear deformation theory, various boundary conditions.

1. INTRODUCTION

Composite materials have interesting properties such as high strength to weight ratio, ease of fabrication, good electrical and thermal properties compared to metals. A laminated composite material consists of several layers of a composite mixture consisting of matrix and fibers. Each layer may have similar or dissimilar material properties with different fiber orientations under varying stacking sequence [1]. There are many open issues relating to design of these laminated composites.Design engineer must consider several alternatives such as best stacking sequence, optimum fiber angles in each layer as well as number of layers itself based on criteria such as achieving highest natural frequency or largest buckling loads of such structure [2]. Analysis of such composite materials starts with estimation of resultant material properties.

Laminated composite materials are extensively used in aerospace, defense, marine, automobile, and many other industries. They are generally lighter and stiffer than other structural materials. A laminated composite material consists of several layers of a composite mixture consisting of matrix and fibers [3]. Each layer may have similar or dissimilar material properties with different fiber orientations under varying stacking sequence. Because, composite materials are produced in many combinations and forms, the design engineer must consider many design alternatives. It is essential to know the dynamic and buckling characteristics of such structures subjected to dynamic loads in complex environmental conditions [4]. For example, when the frequency of the loads matches with one of the resonance frequencies of the structure, large translation/torsion deflections and internal stresses occur, which may lead to failure of structure components. The structural components made of composite materials such as aircraft wings, helicopter blades, vehicle axles and turbine blades can be approximated as laminated composite beams. The use of composites as engineering materials has increased greatly in recent years [5]. These physical properties thus need to be known in order to be analyzed and structurally designed. Having a high strength/weight ratio, these materials are produced through a range of plys, which have orthotropic features, one on top of another, with different angles and forms. Their mechanical properties are thus related to their forms and angles [6].Composite materials are gaining popularity because of high strength, low weight, resistance to corrosion, impact resistance, and high fatigue strength [7]. Other advantages include ease of fabrication, flexibility in design, and variable material properties to meet almost any application.

2. ANALYSIS OF COMPOSITE BEAMS

2.1 Configuration

The material, boundary condition and orientation angle used for composite beam is described in Table 3.1 (i.e) each material must be solved with four boundary conditions and each boundary condition must be with four angle orientation (i.e) each material has 16 models.

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Table 1 Configuration of Composite beam

Materials	Boundary Conditions	Angle orientation
Glass – Epoxy	Clamped – Clamped	0°/0°/0°/0°
Carbon – Epoxy	Clamped – Free	0°/90°/0°/90°
Glass – Polyester	Clamped – Simply supported	0°/45°/0°/45°
Graphite – Epoxy	Simply supported – Simply supported	+45°/-45°/+45°/-45°

The materials used for composite beam has been tabulated for the properties of composite beam as given in Table 2, and the material properties of composite beams are collected from the available literatures.

Table2 Properties of composite beams

Composite Laminates	E1 (GPa)	E2 (GPa)	υ 12	G12 (GPa)	G23 (GPa)	G13 (GPa)	ρ (kg/m3)
Glass - Polyester	37.41	13.67	0.3	5.478	6.03	6.666	1968.9
Graphite -Epoxy	144.80	9.65	0.3	4.14	3.45	4.14	1389.23
Carbon- Epoxy	170	15	0.41	6.2	4.2	6.2	1510
Glass- Epoxy	1603.57	219.83	0.37	117.11	131.89	117.11	1548

3. MODELLING

The composite beam with the structural configuration shown in the Fig.1 has the length of 300mm, width of 25mm and 1.25mm thickness of each layer (i.e. 4 layer). Modal analysis of ANSYS is used to determine the natural frequencies and mode shapes, which are important parameters in the design of a structure for dynamic loading conditions. They also required for spectrum analysis or for a mode superposition harmonic transient analysis. Modal analysis in ANSYS program is linear analysis. The mode extraction method includes Block Lanczos (default), sub space, Power Dynamics, reduced, asymmetric, and damped and QR damped. The damped and QR damped methods allow to include damping in the structure.The ANSYS procedure is given to determine the natural frequencies of different composite beams. Structural data which are required for simple vibration analysis of different composite beams on ANSYS Graphical User Interface (GUI) mode areYoung's modulus, Shear modulus, Poisson's ratio, Density, Length, Breadth and Height.
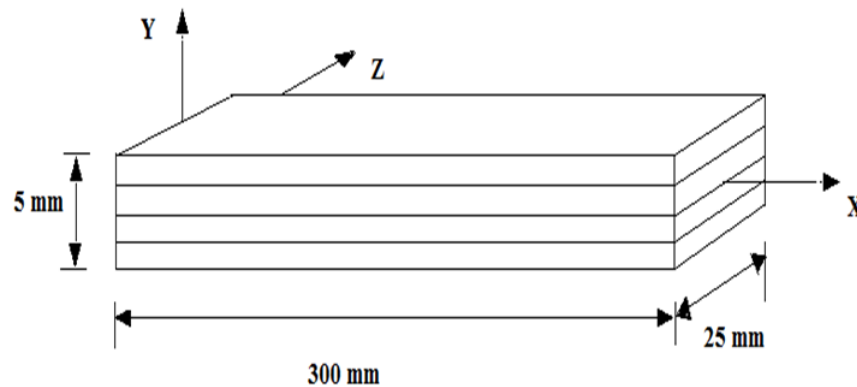

Figure 1 Structural configuration of composite beam

4. ANALYSIS OF BEAMS

ANSYS is a general purpose finite element modeling software for numerically solving a wide variety of engineering problems includes: static/dynamic structural analysis (both linear and nonlinear), heat transfer and fluid problems, as well as acoustic and electro-magnetic problems [8].

It also used to compute elastic-plastic, thermal stress, buckling and vibration analysis of the laminated composite beams.

The shell 281 element type was selected for 3-D modeling of solid structures in ANSYS 14.5. Initially, the beams are modeled in order to get initial estimation of the undamped natural frequencies and mode shape n. The beam is discretized using twenty finite elements; type of element used was SHELL 281(Fig. 2). This element has 8 nodes and it is constituted by layers that are designated by numbers (LN – Layer Number), increasing from the bottom to the top of the laminate [9]. The element has six degrees of freedom at each nodes (i.e) translations in the nodal x, y and z directions and rotation about the nodal x,y and z axes. The model of composite beam is generated with four layers [10].
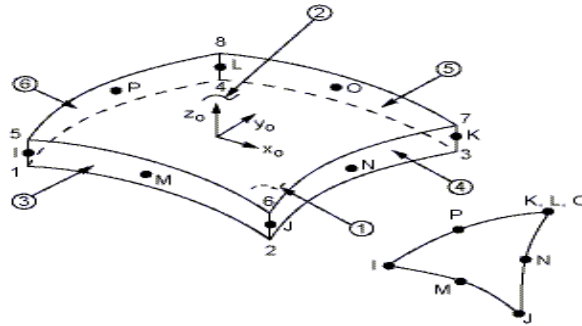
Fig.2. SHELL 281

The material properties were then entered in the program, and the constraint imposed to stimulate for various boundary condition of composite beam. After meshing process is generated, a glass – epoxy composite beam has 400 elements and 1281 nodes. The glass – epoxy composite beam was modeled and the natural frequencies are compared. The Fig. 3 shows number of layers and fibre angle orientation of each layer of composite beam.

5. FIRST ORDER SHEAR DEFORMATION THEORY

Consider a composite beam made of N perfectly bonded layers of different materials or the same fibre-reinforced composite material with different orientations and with different boundary conditions.

The displacement field equations of the composite beam are assumed as follow:

$u(x,y,z)= u0(x,y)-z[c0\ w,x + c1\ \varphi(x)] + c2\ z2\ \psi(x) + c3\left(\frac{z}{h}\right)3\ [\varphi(x)+ w,x]$ (1a)

$v(x,y,z)= v0(x,y)-zw,y$ (1b)

$w(x,y,z)= w0(x,y)$ (1c)

where,

u,v & w are displacements filed equations along the coordinates of x,y & z,

u0, w0 are displacement of a point (x,y,z) at the mid plane,

$\varphi(x)$ & $\psi(x)$ are the rotational angles of cross section.

By selecting the constant values of equation (1a) as: c0=0, c1=1, c2=0, c3=0, the displacements field equation for first order shear deformation theory at any point through the thickness can be expressed as:

$u(x,y,z)= u0(x,y)-z\ \varphi(x,y)$

$v(x,y,z)= v0(x,y)-zw,y$

$w(x,y,z)= w0(x,y)$ (2)

The strain at a point (x,y,z) are given by

$\varepsilon x = u0,x – zw,xx = \varepsilon\ x0\ + z\ \kappa x0$ (3a)

$\varepsilon y = v0,x – zw,yy$

$\gamma xy = -\varphi x + w\ ,0x = \gamma xy\ 0$ (3b)

where,

u0,x , v0,x and (u0,x + v0,x) are the mid-plane normal and shear strain respectively, denoted by $\varepsilon\ 0x$, $\varepsilon\ 0y$ and $\gamma\ 0xy$.

Total strains in short form:

$\{\varepsilon\}x\text{-}y=\{\varepsilon0\}+z\{\kappa\}$ (4)

where, $\{\varepsilon0\}$ & $\{\kappa\}$ approximately defined mid-plane strain and curvature vectors.

If the strains are known at any point along the thickness of the laminate, the stress-strain equation can be given as,

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix}$$

$(or)\{\sigma\}x\text{-}y(k)=[\bar{Q}_{ij}](k)\ \{\varepsilon\}x\text{-}y$ (5)

0°/0°/0°/0°
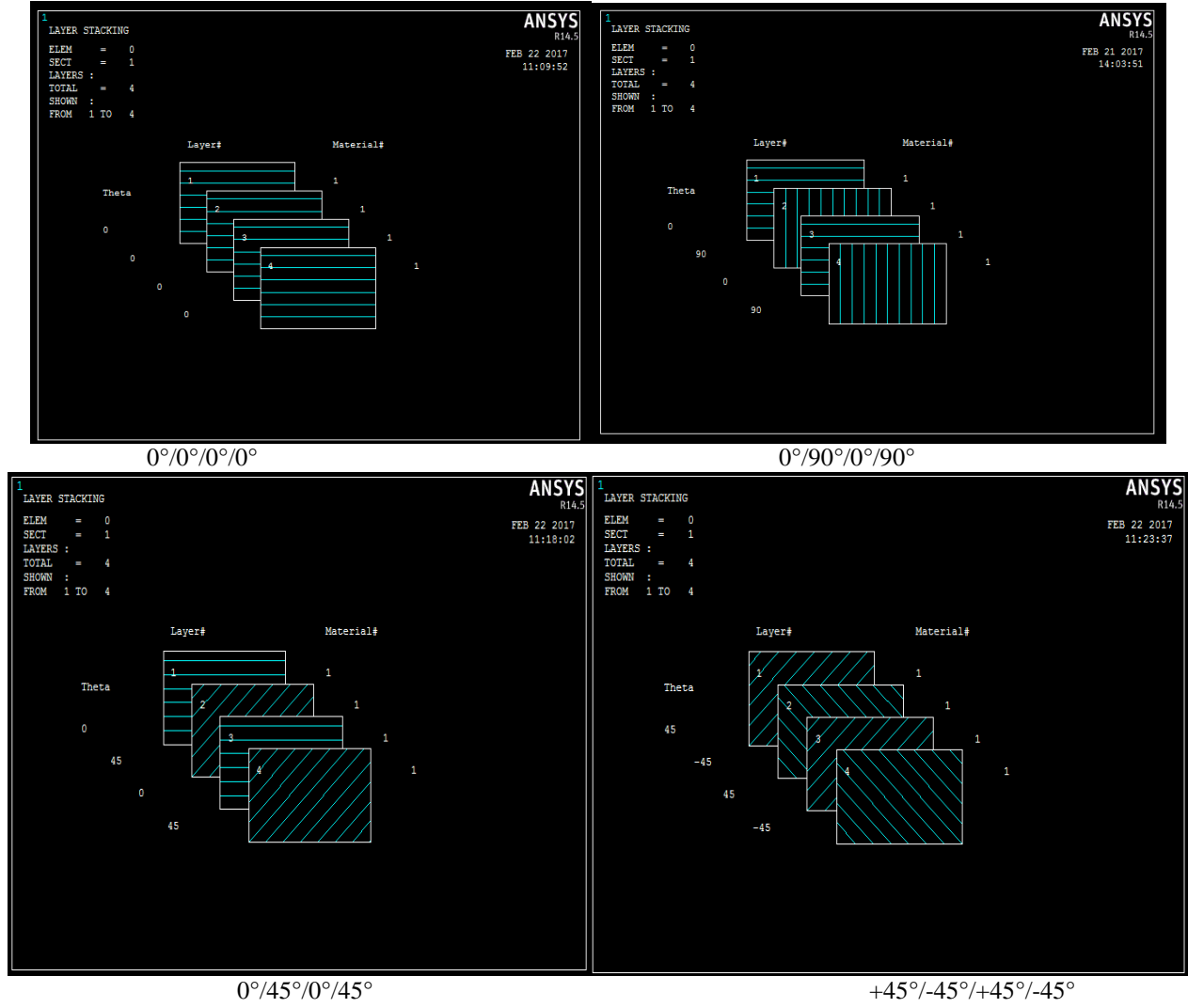

0°/90°/0°/90°

0°/45°/0°/45°

+45°/-45°/+45°/-45°

Fig. 3.Angle orientation of composite beam

The reduced transformed stiffness matrix corresponds to that of the ply located at the point along the thickness of the laminate. Substitute equation (5) in (4).

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} + z \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

(6)

From equation (6) the stresses vary linearly only through the thickness of each lamina. The stresses however may jump from lamina to lamina because the transformed reduced - stiffness matrix changes from ply to ply because depend on the material and orientation of the ply.

These stress resultant are given by

$$(Nx, Ny, Nxy) = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x, \sigma_y, \tau_{xy}) dz}{} = \sum_{k=1}^{n} \int (\sigma_x, \sigma_y, \tau_{xy}) dz$$

(7)

$$(Mx, My, Mxy) = \frac{\int_{-\frac{h}{2}}^{\frac{h}{2}} (\sigma_x, \sigma_y, \tau_{xy}) z dz}{} = \sum_{k=1}^{n} \int (\sigma_x, \sigma_y, \tau_{xy}) z dz$$

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \sum_{k=1}^{n} \int_{h_{k-1}}^{h_k} \begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix}_k z \, dz$$

(8)

Substituting equation (8) in (6) we get

$$\begin{bmatrix} N_x \\ N_y \\ N_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} dz \right\} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} +$$

$$\left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z\,dz \right\} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

(9a)

$$\begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix} = \left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z\,dz \right\} \begin{bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{bmatrix} +$$

$$\left\{ \sum_{k=1}^{n} \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{bmatrix}_k \int_{h_{k-1}}^{h_k} z^2\,dz \right\} \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

(9b)

and substituting the above equation in (9), gives

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

(10a)

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}$$

(10b)

The reduced constitutive matrix,

$$\begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix} = \begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{Bmatrix} \{\varepsilon^0\} \\ \{\kappa\} \end{Bmatrix}$$

(11)

The transformed reduced stiffness constants $\overline{Q_{ij}}$ (i,j=1,2,6) are given by,

$$Q11 = \frac{E_1}{1-v_{12}v_{21}}, \quad Q12 = \frac{v_{12}E_2}{1-v_{12}v_{21}} \quad Q22 = \frac{E_2}{1-v_{12}v_{21}} \quad Q66 = G12$$

(12)

$\bar{Q}11 = Q11\, c4 + Q22\, s4 + 2\, c2\, s2\ (Q12+2Q66)$

$\bar{Q}22 = Q11\, s4 + Q22\, c4 + 2\, c2\, s2\ (Q12+2Q66)$

$\bar{Q}12 = Q12\ (c4 + s4) + c2\, s2\ (Q11+ Q22 -4Q66)$

$\bar{Q}66 = Q66\ (c4 - s4) + c2\, s2\ (Q11+ Q22 -2Q12-2Q66)$

$\bar{Q}16 = (Q11 -Q12 -2Q66)\, c3\, s + (Q12 -Q22 +2Q66)\, s3\, c$

$\bar{Q}26 = (Q11 -Q12 -2Q66)\, s3\, c + (Q12 -Q22 +2Q66)\, c3\, s$

(13)

VIBRATION OF COMPOSITE BEAM

On a structure dynamic loading can vary from recurring cyclic loading of the same repeated magnitude, such as a unbalanced motor which is turning at a specified number of revolutions perminute on a structure (for example), to the other extreme of a short time, intense, nonrecurringload, termed shock or impact loading, such as a bird striking an aircraft component during flight. A continuous infinity of dynamic loads exists between these extremes of harmonic oscillationand impact associated mode shapes. Mathematically, there are infinity of natural frequenciesand mode shapes in a continuous structure.

Dynamic loading can vary from intense, nonrecurring load known as shock load such as bird striking aero plane to recurring cyclic loading of magnitude which repeats itself such as unbalanced motors rotating at particular R.P.M. Any structures amplitude may rapidly grow with time if its frequency of oscillation matches its natural frequency. Structure can be overstressed which leads to its failure or due to large oscillations amplitude may be limited at large value which further leads to fatigue damages. Time dependent loading should be compared with natural frequency to ensure structural integrity of any structure. These two frequencies should be considerably different. While designing structure over deflecting and overstressing should be taken care of and resonances should be avoided.

ω_n is the natural circular frequency in radians per unit time for the nth vibrational mode. Note that in this case there is one natural frequency for each natural mode shape, for n = 1, 2, 3, ..., etc. ω_n can be expressed as,

$$\omega_n = \frac{n^2 \pi^2}{L^2} \sqrt{\frac{b \cdot D11}{\rho \cdot A}}$$

(14)

Transverse-shear effect is not taken into consideration in this equation. For each n there would be different natural frequency which canbewrittenas,

$$\omega_n = \alpha^2 \sqrt{\frac{b \cdot D_{11}}{\rho \cdot A \cdot L^4}}$$

(15)

Frequency in hertz can be determined by,

$$f_n = \frac{\omega_n}{2 \cdot \pi}$$

(16)

The values of α^2 have been catalogued by Warburton, Young and Felgar. The natural frequency of a free-free supported beam is equal to natural frequency of clamped-clamped supported beam. Natural frequencies would be lower if transverse shear deformation effects were included.

Table 4.1 Tabulation of α² values for use in determining the Natural Frequencies

n	Clamped – Clamped	Clamped – Free	Clamped- Simply Supported	Simply Supported- Simply Supported
1	22.37	3.516	15.42	9.870
2	61.67	22.03	49.96	39.48
3	120.90	61.70	104.25	88.83

6. RESULTS AND DISCUSSION
ANSYS 14.5 software package and shear deformation theory are used for the results given below.
Natural frequencies obtained from this ANSYS and shear deformation theory are listed in tables and those results compared for the various composite beams with different boundary conditions and different orientation angles. And mode shapes are presented by ANSYS for different boundary conditions and different orientation angle. Therefore, the results obtained from the analysis are compared. In this chapter, numerical examples are taken to analysis the natural frequencies and mode shapes of the composite laminated beam
In this section the mathematical formulation requires same material properties as ANSYS inputs such as (E1, E2, E3, G12, G23, ρ, υ12 and υ21). This is because of plane stress assumption. In this chapter, the results are also obtained for the first order shear deformation theory and the results are compared. SHELL 281 elements were considered for the analysis of composite beams on the ANSYS 14.5.

7. COMPARISON OF RESULTS
The comparison of results gives a good agreement and hence the error in natural frequency is determined by,

$$\text{Error}(\%) = \left(\frac{Analytical\ values - FSDT\ values}{Analytical\ values} \right) * 100$$

Table 5.9 Comparison of Natural frequency of GLASS – EPOXY composite beam

0/0/0/0	1			2			3		
	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR
C-C	217.16	214.73	1.118	597.59	589.88	1.290	1057.1	1156.43	-9.396
C-F	34.153	33.63	1.531	171.17	167.89	1.916	213.83	597.84	-179.59
C-S	149.68	147.49	1.463	171.17	169.8	0.800	484.4	480.3	0.846
S-S	0	0	0	0	0.0003	0	0.059	0.053	10.169

0/90/0/90	1			2			3		
	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR
C-C	417.96	420.76	-0.669	1144.6	1137.7	0.602	1784.8	1769.45	0.86
C-F	65.983	61.76	6.4	345.03	342.8	0.646	412.16	408.7	0.839

| C-S | 288.64 | 283.78 | 1.683 | 345.12 | 341.98 | 0.909 | 930.78 | 926.56 | 0.453 |
| S-S | 0 | 0 | 0 | 0.027 | 0.018 | 33.33 | 0.046 | 0.038 | 17.39 |

0/45/0/45	1			2			3		
	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR
C-C	252.93	250.68	0.889	695.07	693.43	0.235	1235.6	1233.87	0.140
C-F	31.872	30.47	4.398	159.84	156.63	2.008	199.4	195.34	2.036
C-S	175.36	173.54	1.037	197.75	193.65	2.073	560.85	560.32	0.094
S-S	0	0	0	0	0	0	0.0097	0.0092	5.154

45/-45/ 45/-45	1			2			3		
	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR	ANSYS	FSDT	ERROR
C-C	292.97	290.34	0.8977	805.93	803.23	0.33502	1415.5	1411.32	0.2953
C-F	11.228	10.23	8.88849	54.985	52.45	4.61035	70.245	65.89	6.19973
C-S	199.23	198.26	0.48687	225.44	222.23	1.42388	647.16	641.45	0.88232
S-S	0	0	0	0.028	0.024	14.2857	0.059	0.051	13.5593
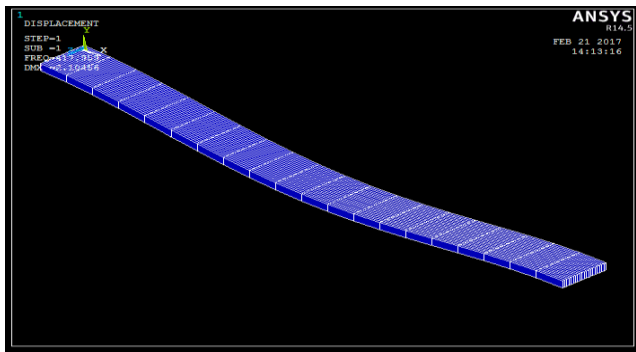
4a) Mode 1- 1st Flexural mode   (4b) Mode 2- 2nd Flexural mode
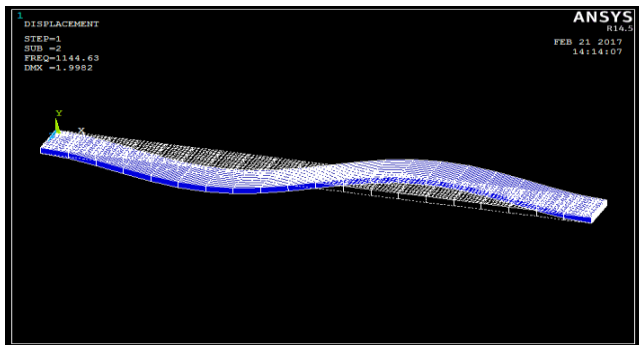
8. CONCLUSIONS

Based on finite element modelling and first order shear deformation theory, the following conclusions are drawn.

The natural frequency is higher in glass-epoxy composite beam.

The natural frequency changes with the change in orientation angle.

The natural frequency is minimum for simply – simply supported beam and maximum for clamped-clamped boundary condition of composite beam.

The largest natural frequency is obtained for clamped-clamped boundary condition with fibre orientation of [0°/90°/0°/90°].

The natural frequency has occurred close agreement between finite element modelling and first order shear deformation theory.

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